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The following summary of results is appended.

Effect of meat ingestion on hourly metabolism

EXP. NO.	DATE	FOOD	NUMBER OF HOURS	URINE N	R. Q.	CALORIES		C DEPOSITED	R. Q. OF DEPOSIT	HOURS AFTER FOOD
						Indirect	Direct			
	1919									
27	Feb. 6	Basal	2	0.15	0.84	15.92	16.08			(Weight = 11.24 kgm.)
29	Feb. 17	Meat, 700 g.	3	1.40	0.820	29.97	30.01	0.60	0.73	4, 5, 6
30	Feb. 18	Meat, 800 g.	3	1.46	0.800	31.47	32.75	0.66	0.83	4, 5, 6
31	Feb. 19	Meat, 900 g.	3	1.47	0.787	34.33		0.48	0.96	4, 5, 6
32	Feb. 20	Meat, 1000 g	2	1.46	0.797	34.27	34.50	0.42	0.87	4, 5
			2	1.46	0 820	35.50	37.20	0.23	0.62	6, 7
33	Feb. 21	Meat, 1100 g.	2	1.45	0.831	31.65	31.36	0.56	0.68	4, 5
			2	1.45	0.843	35.28	34.54	0.25	0.49	6, 7
34	Feb. 24	Meat, 1080 g.*	2	1.57	0.800	34.00	34.12	0.70	0.83	4, 5
35	Feb. 26	Basal	3	0.27	0.82	19.74	19.59			(Weight = 12.07 kgm.)
36	Feb. 27	Basal	3	0.20	0.83	18.25	17.16			
37	Feb. 28	Basal	2	0.17	0.85	17.30	16.95			
38	Mar. 1	Basal	2	0.15	0.82	18.21				
39	Mar. 3	Basal	3	0.15	0.85	17.57	17.22			
43	Mar. 12	Basal	2	0.15	0.81	17.08	16.99	0		(Weight = 11.50 kgm.)
46	Mar. 17	Meat, 1200 g.	3	1.02	0.796	26.57	28.10	0		5, 6, 7 after 1 day's fast
47	Mar. 18	Meat, 800 g.	3	1.44	0.795	29.90	30.77	0.77	0.84	5, 6, 7
48	Mar. 19	Meat, 800 g.	4	1.35	0.793	29.37	30.27	0.61	0.86	5 to 8
49	Mar. 22	Basal	2	0.23	0.79	17.72	17.54			
50	Mar. 24	Basal	2	0.16	0.84	17.26	16.87			
51	Mar. 28	Meat, 800 g.	4	1.02	0.795	27.04	27.52			5 to 8 after 4 days' fast
54	Apr. 15	Meat, 800 g.	4	1.41	0.794	31.07	30.57	0.59	0.86	5 to 8
55	Apr. 16	Meat, 1000 g.	4	1.58	0.797	31.97	31.98	0.91	0.83	5 to 8
56	Apr. 19	Meat, 1300 g.	4	1.47	0.826	31.62	33.25	0.59	0.71	5 to 8 after 1 day's fast

* Standard diet at 5 p.m. and thereafter daily until March 15.

ON THE TWIST IN CONFORMED MAPPING

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Communicated by E. H. Moore, April 29, 1919

Note II on Conformal Mapping under aid of Grant No. 207 from the Bache Fund.

Let $w = w(z)$ be a power series in z, convergent for $|z| < 1$ and such that the circle $|z| < 1$ is mapped conformally on a *simple* (that is, simply connected and nowhere overlapping) region in the w-plane. By a linear transformation $w^1 = aw + b$, we may reduce $w(z)$ to the form $z + a_2z^2 + \ . \ . \ . + a_nz^n +$

. . . . For the comparison of maps of different regions, two geometrical concepts are of fundamental importance: the *distortion* = $|dw/dz|$, or the ratio of the lengths of corresponding line elements in the w- and z-planes, and the *twist* = imaginary part of log (dw/dz), or the angle between corresponding line elements, this angle being always taken between $-\pi$(excl.) and π (incl.). Koebe[1] has shown that on the circle $|z| = r$, $0 < r < 1$, both $|dw/dz|$ and $|z|$ lie between positive bounds which depend on r alone, and the writer[2] has determined the exact values of these bounds.

It is the purpose of the present note to state the corresponding result in regard to the twist:

When the analytic function

$$w = z + a_2 z^2 + \ . \ . \ . + a_n z^n + \ . \ . \ .$$

maps the circle $|z| < 1$ *on the interior of a simple region D in the w-plane, the twist* τ *satisfies the following inequalities for* $|z| = r$ *and* $0 < r < 2^{-\frac{1}{2}}$

$$-4 \arcsin r < \tau < 4 \arcsin r, \tag{1}$$

except when

$$w = \frac{z\,(1 - \cos\beta \cdot e^{\alpha i} z)}{(1 - e^{(\alpha+\beta) i} z)^2}, \tag{2}$$

where α *and* β *are real, and* $\cos\beta = r$. *In this case,* τ *attains the upper or lower bound in* (1) *for* $z = re^{-\alpha i}$ *according as* $\beta = +\operatorname{arc\,cos} r$ *or* $\beta = -\operatorname{arc\,cos} r$. *For* $2^{-\frac{1}{2}} \leqq r < 1$, *we have only the inequality included in the definition of* τ

$$-\pi < \tau \leqq \pi, \tag{3}$$

and no single class of functions analogous to (2) *reaching the upper and lower bounds can be assigned.*

When the region D is CONVEX, *we have for* $|z| = r$ *in the whole interval* $0 < r < 1$

$$-2 \arcsin r < \tau < 2 \arcsin r \tag{4}$$

except when

$$w = \frac{z}{1 - e^{(\alpha+\beta) i} z}, \quad \cos\beta = r, \tag{5}$$

the upper and lower bounds being then attained as above.

The proof is similar to that of the distortion theorem outlined before.[2] The region D is approximated by rectilinear polygons, for which we have the formula of Schwarz

$$\frac{dw}{dz} = (1 - e^{\alpha_1 i} z)^{\mu_1} (1 - e^{\alpha_2 i} z)^{\mu_2} \ . \ . \ . \ (1 - e^{\alpha_m i} z)^{\mu_m} \tag{6}$$

whence, for $z = re^{\theta i}$,

$$\tau = \Sigma_1^m - \mu_\nu \arctan \frac{r \sin(\theta + \alpha_\nu)}{1 - r\cos(\theta + \alpha_\nu)} + 2k\pi i, \tag{7}$$

where the arctangents are all taken between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$, and the integer k is chosen so as to make $-\pi < \tau \leqq \pi$. Since the polygon is not overlapping, we have

$$\Sigma\mu_\nu = -2, \; -3 \leqq \mu_\nu \leqq 1 \quad (\nu = 1,2, \ldots, m)$$

For a fixed value of m, we consider the set of all the values of $\alpha_1, \ldots, \alpha_m$, $\mu_1, \ldots, \mu_m$ for which (6) maps $|z| < 1$ on a *simple* polygon, and it is first shown that this set is *closed*. On this set, and for $-\pi \leqq \theta \leqq \pi$ the expression (7) has therefore a maximum and a minimum, which are found by observing that

$$\arctan \frac{r \sin \varphi}{1 - r \cos \varphi}$$

has a maximum $=$ arcsin r for $\varphi =$ arccos r and a minimum $= -$ arcsin r for $\varphi = -$ arccos r. In the discussion of (7), it is necessary to distinguish the case where all μ are negative, corresponding to a convex polygon, from the general case where some μ are positive; for this reason, the convex regions appear separately in the statement of the theorem. By considerations of continuity, and the use of elementary properties of harmonic functions, it is finally shown that the upper and lower bounds in (1) and (4) are reached in the cases (2) and (5) only, and that no closer bounds than (3) can be found for $2^{-\frac{1}{2}} \leqq r < 1$ in the non-convex case.

Regarding (2) and (5), we observe that by rotating the z- and w-planes through the angle $-\alpha$ about their origins, we may make $\alpha = 0$; in this case, the circle $|z| < 1$ is mapped by (2) on the w-plane slit along the straight line segment

$$w = \frac{i}{4 \sin \beta} - i e^{-2\beta i} t, \qquad (0 \leqq t \leqq + \quad),$$

and by (5) on the half plane in which the real part of $we^{\beta i}$ is greater than $-\frac{1}{2}$.

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[1] Koebe, *Göttingen, Nachr. Ges. Wiss.*, 1909, (73).

[2] Gronwall. *Paris, C. R. Acad. Sci.*, 162, 1916, (249).

SEVENTEEN SKELETONS OF MOROPUS; PROBABLE HABITS OF THIS ANIMAL

By Henry Fairfield Osborn

American Museum of Natural History, New York City

Read before the Academy, April 29, 1919

Moropus is an aberrant perissodactyl, closely related to the family of the Titanotheres and more remotely to that of the Horses. It occurs in the Lower Miocene age in France and North America, and its ancestors have been